# Quantifying the Effect of Demand Response on Electricity Markets

Chua-Liang Su, *Member, IEEE*, and Daniel Kirschen, *Fellow, IEEE*

***Abstract*—It is widely agreed that an increased participation of the demand side in the electricity markets would produce benefits not only for the individual consumers but also for the market as a whole. This paper proposes a method for quantifying rigorously the effect that such an increase would have on the various categories of market participants. A new centralized complex-bid market-clearing mechanism has been devised to take into consideration the load shifting behavior of consumers who do submit price-sensitive bids. The effects of the proportion of demand response on the market are illustrated using a test system with ten generating units scheduled over 24 periods.**

***Index Terms*—Demand response, demand-side participation, electricity markets, global welfare maximization, load shifting, mixed-integer linear programming.**

## I. Introduction

IN most electricity markets, consumers play a much more limited role than producers. While there are some good practical reasons for this difference, it is widely acknowledged that a more active participation in the market by the demand side could have significant benefits. In particular [1]–[6]:

- consumers who can shift their load from periods of high prices to periods of lower prices will reduce their energy cost;
- this shifting of demand will flatten the aggregated load profile and hence reduce the overall cost of producing electrical energy;
- consumers who do not adjust their demand in response to prices are therefore also likely to benefit if this reduction in cost translates into a reduction in prices;
- the ability of generating companies to exert market power will be reduced.

On the other hand, a more elastic demand will generally reduce the profits of the generating companies [7], [8]. These consequences are consistent with standard microeconomics theory but have not so far been quantified using a rigorous method. This paper describes a technique for achieving this goal that takes into account the specificities of electricity generation, consumption, and trading.

With regard to consumption, it is essential to model properly how consumers might respond to time-dependent prices for electrical energy. There is some evidence that consumers will temporarily reduce their consumption of electrical energy when they are faced with a sudden and very large increase in price. However, in the long run, when faced with periodic fluctuations in prices, consumers are much more likely to shift their demand in a way that balances potential cost savings against the inconvenience or extra expense that would result from the time shifting of energy consumption. If consumers reduced their demand during periods of high prices, and did not catch up at other times, this would mean that the value they put on electrical energy is not consistent. This means that consumers merely shift some of their demand from one period to another in response to price signals. This paper therefore assumes that the total energy consumption of a price-responsive consumer over one price cycle is independent of the shape of the price profile during that cycle.

If the demand-side is to take an active and significant part in the market for electrical energy, the market-clearing mechanism must accept price-sensitive demand-side bids and take into account the load shifting behavior of consumers. This paper therefore proposes a day-ahead market-clearing mechanism that allows consumers to submit complex bids. These complex bids give consumers the opportunity to specify constraints on their hourly and daily consumptions in the same way as generators can specify the operating constraints on their generating units. Kirschen and Strbac [9] demonstrated the importance of designing a robust and realistic market-clearing mechanism when demand-side bids are allowed. They showed that bids for demand-side load reductions caused price spikes in the now defunct Electricity Pool of England and Wales (EPEW) because these bids were treated just like negative generation. Borghetti *et al.* developed an auction algorithm that implicitly allows demand shifting [10]. However, in this algorithm the periods when the consumers can reduce their load or recover the energy that they did not consume are fixed. This reduces flexibility and unnecessarily complicates the market rules. As the consumers in the model do not contractually own the load reduction "resources," the auction model is most likely to suffer from gaming opportunities. This is because the bidders could claim to have performed load reduction when they actually had no intention to use electricity. Arroyo and Conejo presented an alternative market-clearing tool for achieving maximum social welfare in a pool market [11]. The consumers in this auction model are required to submit bids to purchase energy explicitly. This means that the consumers will contractually own the demand if the bids are accepted. This auction model does not suffer from the gaming problem associated with Borghetti's model. Contreras introduced a multiround auction

Manuscript received September 09, 2008; revised January 14, 2009. First published July 06, 2009; current version published July 22, 2009. This work was supported by the Engineering and Physical Science Research Council (UK) as part of the Supergen Futurenet Project. Paper no. TPWRS-00694-2008.

The authors are with the School of Electrical and Electronic Engineering, University of Manchester, Manchester M60 1QD, U.K. (e-mail: c.su@manchester.ac.uk;d.kirschen@manchester.ac.uk).

Digital Object Identifier 10.1109/TPWRS.2009.2023259

algorithm that allows market participants to modify their bids consecutively until market equilibrium is reached [12]. As the algorithm is performed iteratively, the market prices may oscillate from one iteration to the next. The oscillatory behavior of the solution is solved by choosing proper stopping criteria. This, however, raises concerns about the equity of the model as the stopping criteria are chosen heuristically. Nevertheless, the model can be used as a benchmark to evaluate the performance of traditional single-round auction designs, by comparing the economic efficiency indicators such as social welfare between auction models. All the auction models described above do not provide consumers an effective mechanism to reduce their electricity costs by shifting their demand. This requires the consumers to be flexible with the timing of consumption. This paper proposes such a model, which will allow, for example, a paper mill to reduce costs by stockpiling paper pulp for use in a later process. Similarly, a group of domestic consumers under the guidance of an aggregator would be able to shift demand to periods where day-ahead prices are expected to be lower. The proposed bidding mechanism is also useful in managing the consumers' risk of going unbalanced after the gate closure of a day-ahead market, especially if the day-ahead and balancing market prices are volatile.

Section II describes the proposed market-clearing mechanism while Section III discusses how the effects of demand response can be quantified rigorously. Numerical results illustrating the effect of a significant and active demand response are presented and discussed in Section IV.

## II. Market-Clearing Mechanism

This section describes a market-clearing mechanism that accepts bids from both the demand and supply sides and takes into account the constraints put forward by both sides. It is therefore a day-ahead market with complex bids and offers whose objective is to maximize the social welfare. The market operator thus has to perform a multiperiod optimization to determine the optimal production and consumption schedules as well as the market-clearing price $\pi^t$ at each period. It is assumed that the marginal generating unit is used to clear the market. A "side-payment" is added to the marginal cost of that generating unit to determine the market-clearing price $\pi^t$. This side-payment allows the marginal generating units to recoup no-load cost (a fixed cost that is incurred by a generating unit regardless of its production level) and start-up cost (a fixed cost that is incurred by a generating unit when it is synchronized). This encourages generators to bid at their actual cost. As these fixed costs are assumed to be zero in the later examples of this paper, the formulation of side payment and its consequence on market-clearing results will not be presented in this paper but is addressed in [8].

Participating in a day-ahead market gives consumers the opportunity to adjust their activities (e.g., their industrial production schedule) once the market has cleared. The number of consumers who could respond substantially to price signals issued closer to real time is probably much smaller. For the sake of simplicity, it is assumed that congestion in the transmission network can be ignored and that ancillary services such as spinning reserve are traded in a separate market. If the production or consumption of a market participant deviates from the amount allocated through this optimization process, the difference is settled in a balancing market, which is cleared separately from the day-ahead market.

### A. Objective Function

As mentioned above, the objective is to maximize the social welfare, i.e., the difference between the value that consumers attach to the electrical energy that they buy and the cost of producing this energy. Mathematically

$$\max \sum_{t=1}^{T} \left(GS^t - OC^t\right) \tag{1}$$

where $GS^t$ and $OC^t$ are, respectively, the consumer gross surplus and the system operating cost at period $t$, and $T$ is the number of periods in the optimization horizon. If the generators or the consumers do not bid at their respective marginal costs or benefits, the objective function is not the social welfare but the "perceived" social welfare [11].

### B. Generators' Offers

Generators submit complex bids that embody not only their operational cost data but also their operational constraints. The operating cost includes the no-load cost, the running cost and the start-up cost. To make possible the solution of this unit commitment problem using a mixed-integer linear programming package [13], piecewise linear cost curves are used:

$$OC^t = \sum_{i=1}^{I} \left( u^{i,t} N^i + \sum_{b=1}^{B} MC^{i,b} P_{Sg}^{i,b,t} + S^{i,t} \right) \tag{2}$$

where

| | |
|---|---|
| $I$ | number of generating units; |
| $B$ | number of segment in the generator's offer curve; |
| $u^{i,t}$ | status of generating unit $i$ at period $t$ (0: on, 1: off); |
| $N^i$ | no-load cost of generating unit $i$; |
| $MC^{i,b}$ | marginal production cost of generating unit $i$ on segment $b$ of its piecewise linear cost curve; |
| $P_{Sg}^{i,b,t}$ | output of generating unit $i$ on segment $b$ of its piecewise linear cost curve during period $t$; |
| $S^{i,t}$ | start-up cost of generating unit $i$ at period $t$. |

The up and down ramping rates of the generators [14] as well as the minimum up and down time constraints [15] are not considered in this paper. However, their impact on the results is analyzed in [8].

### C. Demand-Side Bids

Not all consumers have the ability or the motivation to adjust their demand as a function of prices. Part of the demand will therefore remain perfectly inelastic. Fig. 1 shows the shape of the demand curve that has been adopted for this study.

Equation (3) shows how the consumer gross surplus is calculated based on the accepted demand-side bids and the mar-

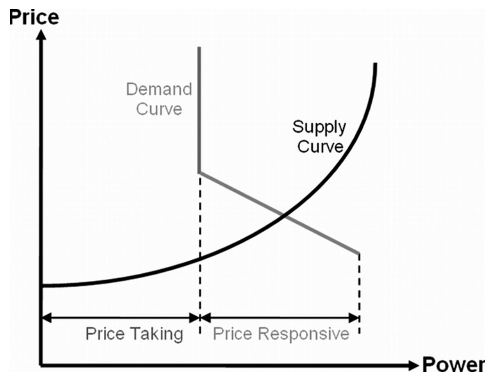


Fig. 1. Price taking and price responsive demand.

ginal value that consumers attach to these bids. This summation is limited to the price responsive part of the demand because the price-taking part has, in theory, an infinite marginal value as otherwise the consumers would have placed price responsive bids with a finite marginal value attached. To avoid the obvious problem that this would cause, the consumer gross surplus for the price-taking part of the demand is assumed constant and hence taken out of the optimization:

$$GS^t = \sum_{k=1}^{K}\sum_{j=1}^{J} MB_{Sg}^{k,j,t} \cdot D_{Sg}^{k,j,t} \tag{3}$$

where

| | |
|---|---|
| $MB_{Sg}^{k,j}$ | marginal benefit of segment $j$ of the bid of demand-side bidder $k$; |
| $J$ | number of segments of the bid of bidder $k$; |
| $K$ | total number of demand-side bidders. |

This model allows consumers to purchase a certain amount of energy ($D_T^{z,t}$, where $z$ is an index of price taking bidders) regardless of the market-clearing prices. The energy for which consumers are price-takers is delivered in the specified volumes for each hour of the scheduling horizon. Consumers who have the means to reschedule their consumption (e.g., using storage devices [8]) can submit bids for energy that are sensitive to electricity prices. The bids that active demand-side participants can submit are quite flexible. In particular, these bids can combine the following characteristics:

- conventional price-volume bid at a specific period;
- minimum energy consumption at any period;
- maximum energy consumption at any period;
- total energy consumption over the scheduling horizon;
- price taking bid for meeting an inflexible demand.

In the optimization program used for market-clearing, these bid specifications are translated into constraints on the demand at each period (4) and on the total demand over the optimization horizon (5). This last specification is implemented as an inequality rather than an equality constraint because a demand-side bid below the lowest price at which generators are willing to produce would otherwise prevent the market from clearing. The set of (6) implements the price responsive bids in a form suitable for mixed-integer linear programming:

$$v^{k,t} \cdot D_L^{k,t} \le D^{k,t} \le v^{k,t} \cdot D_U^{k,t} \tag{4}$$

$$0 \le \sum_{t=1}^{T} D^{k,t} \cdot \Delta t \le E^k \tag{5}$$

$$\left.\begin{array}{c} D^{k,t} - \sum_{j=1}^{J} D_{Sg}^{k,j,t} = 0 \\ v^{k.t} \cdot D_L^{k,t} - D_{Sg}^{k,1,t} \le 0 \\ D_E^{k,0} = 0 \\ 0 \le D_{Sg}^{k,j,t} \le v^{k.t} \cdot \left(D_E^{k,j} - D_E^{k,j-1}\right) \end{array}\right\} \tag{6}$$

where

| | |
|---|---|
| $v^{k,t}$ | status of bid of price responsive bidder $k$ at period $t$ (0: accepted, 1: rejected); |
| $D_L^{k,t}$ | minimum amount of MW that can be consumed by bidder $k$ at period $t$; |
| $D_U^{k,t}$ | maximum amount of MW that can be consumed by bidder $k$ at period $t$; |
| $D^{k,t}$ | consumption of bidder $k$ at period $t$; |
| $\Delta t$ | duration of optimization interval; |
| $E^k$ | maximum amount of energy that is required by bidder $k$ over the optimization horizon; |
| $D_{Sg}^{k,j,t}$ | demand of bidder $k$ during period $t$ on segment $j$ of its bid; |
| $D_E^{k,j}$ | upper limit of segment $j$ of the bid of bidder $k$. |

### D. System Constraints

The generation and demand schedule produced by the optimization program must match at each period the amount of power produced by the generators with the price-taking and price-sensitive components of the demand. Equation (7) summarizes this constraint:

$$\sum_{i=1}^{I} P^{i,t} - \sum_{k=1}^{K} D^{k,t} - \sum_{z=1}^{Z} D_T^{z,t} = 0 \tag{7}$$

where

| | |
|---|---|
| $K$ | number of price-responsive demand-side bidders; |
| $Z$ | number of price-taking demand-side bidders; |
| $P^{i,t}$ | production of generating unit $I$ at period $t$; |
| $D^{k,t}$ | consumption of price-responsive bidder $k$ at period $t$; |
| $D_T^{z,t}$ | consumption of price-taking bidder $z$ at period $t$. |

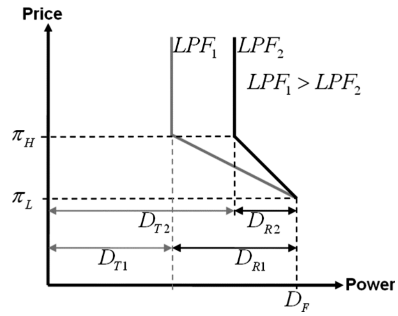


Fig. 2. Relationship between LPF and demand.

## III. Quantifying the Impact of Demand Response

### A. Quantifying the Demand Response

The proportion of the demand that responds to prices affects the shape of the demand curve. Fig. 2 shows how this has been modeled. Considering the parameters of the demand curve shown on this figure, the load participation factor is defined as the ratio of the price responsive demand to the total possible demand:

$$LPF = \frac{D_R}{D_F}. \tag{8}$$

The parameter $D_F$ changes at every period to reflect the natural evolution of the load. On the other hand the load participation factor $LPF$ and the parameters $\pi_L$ and $\pi_H$ remain constant over the scheduling horizon. The price elasticity of the demand is given by [16]

$$\varepsilon = -\frac{\frac{\Delta D}{D}}{\frac{\Delta \pi}{\pi}}. \tag{9}$$

It can then easily be shown that

$$\varepsilon = -LPF \cdot \frac{\pi_L}{\pi_H - \pi_L}. \tag{10}$$

It should be noted that the own price elasticity $\varepsilon$ derived in (10) is not the full demand elasticity if demand shifting is taken into account.

### B. Performance Measure

Because consumers can shift their load from one period to another, demand response affects the profiles of prices and loads over the entire optimization horizon. On the other hand, if the bidding price of a consumer is too low, it may not be possible to shift the corresponding portion of load. The assessment of the benefits of demand shifting must therefore be done taking these facts into account. Conventional economic indicators such as consumer surplus are useful in measuring the total benefits of consumption. However they do not indicate how much benefit is obtained if an additional MWh is consumed. Therefore, this paper proposed the calculation of average prices over the scheduling horizon, weighted by the energy consumed or produced at each period. One could use an average market-clearing price defined as the average of the market-clearing price at each period $\pi^t$:

$$\pi_{AVG} = \frac{1}{T}\sum_{t=1}^{T} \pi^t. \tag{11}$$

However, because of demand shifting, the cost of an additional MWh of energy for price-responsive bidder $k$ is better represented by the following weighted average:

$$\mu_R = \frac{\sum_{t=1}^{T} \pi^t \cdot D^{k,t}}{\sum_{t=1}^{T} D^{k,t}}. \tag{12}$$

A similar weighted average cost was introduced in [17] and can be generalized as follows:

$$\mu = \frac{\sum_{t=1}^{T} X^t \cdot Y^t}{\sum_{t=1}^{T} Y^t} \tag{13}$$

where $X^t$ represents a series of costs or prices and the weighting factors $Y^t$ are the energy consumed or produced at the corresponding periods

$$X^t \in \left\{\pi^t, \sum_{i=1}^{I}\left(MC^{i,t} + \frac{u^{i,t}\cdot N^i + S^{i,t}}{P^{i,t}}\right)\right\} \tag{14}$$

$$Y^t \in \left\{D_T^{k,t}, D^{k,t}, \sum_{i=1}^{I} P^{i,t}\right\}. \tag{15}$$

Table I defines the various weighted average quantities used in this paper.

| | |
|---|---|
| $\mu_D$ | weighted average marginal cost to all consumers; |
| $\mu_G$ | weighted average marginal cost of generators; |
| $\mu_P$ | weighted average revenue collected by generators; |
| $\mu_R$ | weighted average marginal cost for price-responsive consumers; |
| $\mu_T$ | weighted average marginal cost for price-taking consumers. |

These weighted averages represent the "normalized" effective cost or revenue of 1 MWh for the three groups of participants, i.e., price-taking consumer, price-responsive consumer, and generators.

The benefit or loss that demand response creates for a particular group of participants can be measured by taking the differ-

TABLE I
DEFINITION OF WEIGHTED AVERAGES

| $\mu$ | $X^t$ | $Y^t$ |
|---|---|---|
| $\mu_D$ | $\pi^t$ | $D^{k,t} + D_T^{k,t}$ |
| $\mu_G$ | $\sum_{i=1}^{I}\left(MC^{i,t} + \frac{u^{i,t} \cdot N^i + S^{i,t}}{P^{i,t}}\right)$ | $\sum_{i=1}^{I} P^{i,t}$ |
| $\mu_P$ | $\pi^t$ | $\sum_{i=1}^{I} P^{i,t}$ |
| $\mu_R$ | $\pi^t$ | $D^{k,t}$ |
| $\mu_T$ | $\pi^t$ | $D_T^{k,t}$ |

ence between the weighted average prices or costs without and with demand response:

$$\lambda(LPF) = \mu(LPF = 0) - \mu(LPF). \tag{16}$$

Based on this general definition, the following changes in cost or revenue caused by demand response can be defined for various sets of market participants:

- $\lambda_D$ change in cost for the system demand;
- $\lambda_G$ change in operating cost for the generators;
- $\lambda_P$ change in total revenue for the generators;
- $\lambda_R$ change in cost for the demand-shifting price-responsive bidders;
- $\lambda_T$ change in cost for the price-taking bidders;
- $\lambda_{TA}$ change in social welfare for all the participants;
- $\lambda_{TD}$ change in total benefit for all of the demand-side;
- $\lambda_{TG}$ change in total benefit for all the generators.

Because of the system balance constraint (7), the following relation holds:

$$\mu_D = \mu_P. \tag{17}$$

Therefore

$$\lambda_D = \lambda_P. \tag{18}$$

Equation (18) states simply that the benefit to consumers of demand shifting is equal to the loss of revenue of the generators. The difference in profit for the generators is given by

$$\lambda_{TG} = \lambda_G - \lambda_P. \tag{19}$$

Equation (19) indicates that the generators will profit from demand shifting if the resulting reduction of operating cost is more than the loss of revenue. The total relative benefit obtained by the demand-side (i.e., both the price-responsive and the price-taking consumers) is

$$\lambda_{TD} = \lambda_D. \tag{20}$$

A strict definition of $\lambda_{TD}$ should take into account the consumers' gross surplus. However, this has been omitted from (20) because the marginal benefit of consumption of the price-responsive bidders is arbitrarily large and thus has no real meaning. Furthermore, considering the marginal benefit of consumption into these equations would exaggerate the benefit of demand shifting. This is because the marginal benefit of consumption of the demand shifting bidder is assumed to be zero at $LPF = 0$ because it would then submit a price taking bid instead. The total relative benefit obtained by all the participant groups is obtained by summing (19) and (20):

$$\lambda_{TA} = \lambda_D + \lambda_G - \lambda_P. \tag{21}$$

Substituting (18) gives

$$\lambda_{TA} = \lambda_G \tag{22}$$

which means that demand shifting spreads the reduction in the operating cost of the generators among all the market participants.

## IV. NUMERICAL RESULTS

The market-clearing algorithm described in Section III has been applied to several scenarios to assess the economic viability of demand shifting and evaluate its impact on the market. The test system used in the studies consists of ten generating units with a total capacity of 5545 MW. The maximum and minimum loads are 4400 MW and 1850 MW, respectively, while the total system forecasted demand is 77 095 MWh. Details of this system are provided in the Appendix .

### A. Effect of Demand Shifting

This first test analyzes the impact of demand shifting on the demand-shifting price-responsive bidders, on the price-taking bidders and on the generators. For simplicity, it is assumed that there is only one demand-shifting bidder and one price-taking bidder. The demand-shifting bid is modeled as follows:

$$E^1 = LPF \cdot \sum_{t=1}^{T} D_F^t \cdot \Delta t \tag{23}$$

$$D_U^{1,t} = \frac{E^1}{\Delta t} \tag{24}$$

$$D_L^{1,t} = 0 \tag{25}$$

$$MB_{Sg}^{1,1,t} = \pi_H = \pi_L. \tag{26}$$

Because $\pi_H = \pi_L$, $\varepsilon \longrightarrow -\infty$, this portion of the demand curve is horizontal and the price-responsive part of the demand is

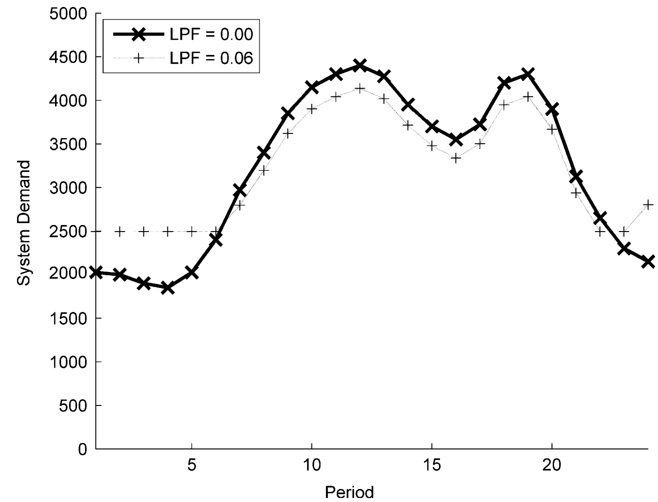


Fig. 3. System demand.

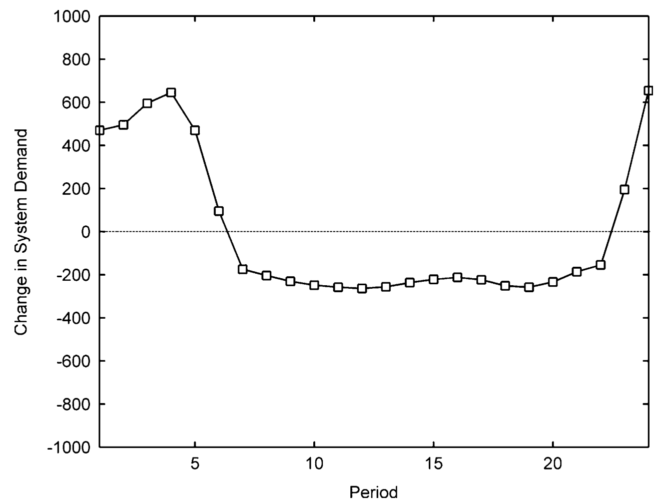


Fig. 4. Change in system demand.

perfectly elastic. In addition, $MB_{Sg}^{k,j,t}$ is given a sufficiently high value (at least equal to the highest marginal cost offer of the most expensive generator, which is 11.057 $/MWh) that the entire energy requirement defined in (23) will definitely be satisfied. This will ensure that the price-responsive demand is shifted across the scheduling horizon in a way that minimizes the system operating cost. (The gross benefit of demand consumption is constant because this portion of the demand curve is flat.) This makes possible a fair assessment of the benefits of demand shifting as demand response $(LPF)$ increases, as otherwise more demand shifting bids may be rejected and this deflates the marginal cost of price responsive demand $(\mu_R)$ since the market-clearing price tends to decrease with the reduction of system demand.

The price-taking demand is modeled as follow:

$$D_T^{1,t} = (1 - LPF) \cdot D_F^t. \tag{27}$$

To facilitate understanding of these numerical results, the generating units' no-load and start-up costs have been omitted in this and subsequent studies.

Figs. 3–6 show the effects of increasing the proportion of price responsive demand on the system load profile and the market-clearing prices $(\pi^t)$. The system demand shifts from high demand periods to fill up the valleys at both ends of the planning horizon when the $LPF$ increases. These results show that, while a reduction in the load generally reduces the market-clearing price in the period where it takes place, the load recovery causes price increases at other periods. For example, the significant increase in the market-clearing price that can be observed at periods 4 and 24 is largely due to a significant demand shift to these periods. On the other hand, the decrease in market-clearing price during the periods of demand reduction is relatively moderate, and for some periods (e.g., $t = 7$ and 16) there is no effect. Although this is not the case here, load recovery does not always increase the market-clearing price. This insensitivity is largely due to the non-convexity of the generators' marginal cost curve.

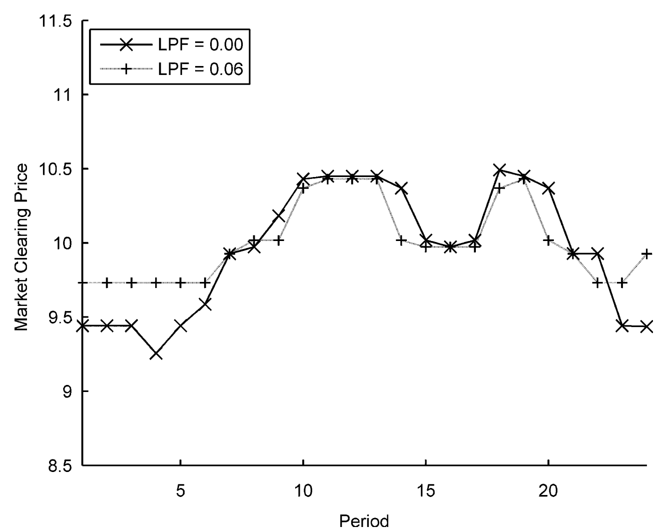


Fig. 5. Market-clearing prices.

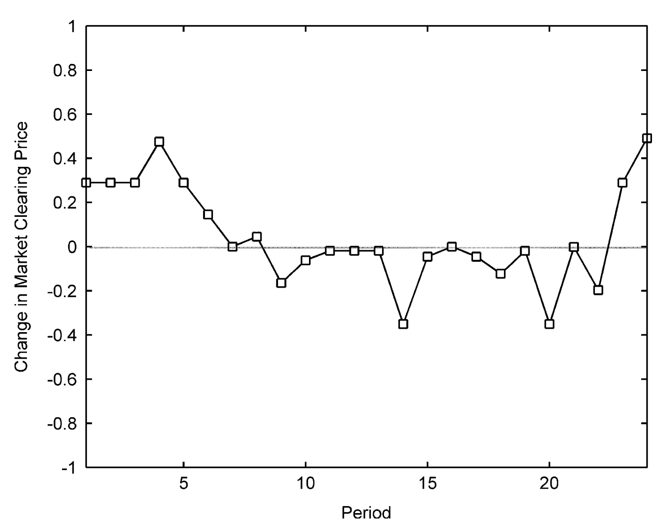


Fig. 6. Change in market-clearing prices.

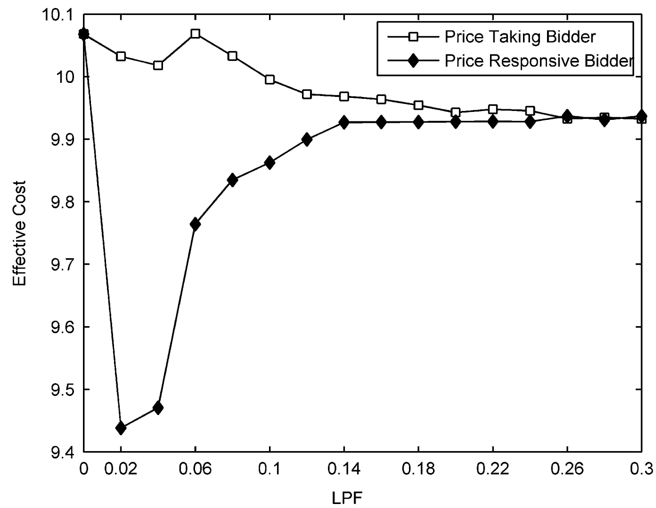


Fig. 7. Effective costs for price-responsive and price-taking demand.

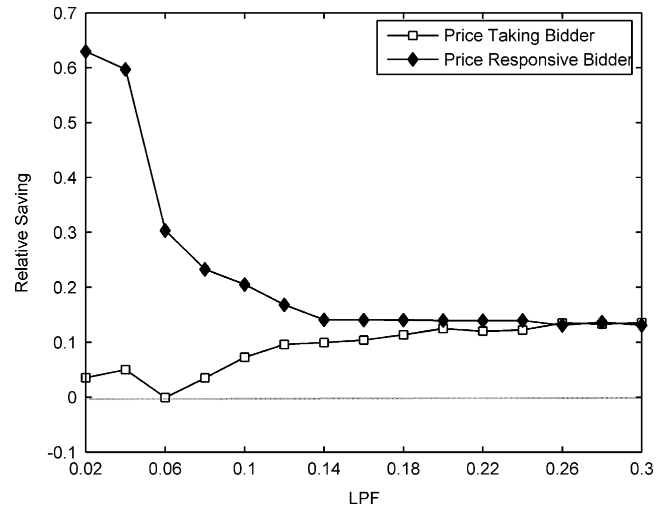


Fig. 8. Savings for price-responsive and price-taking demand.

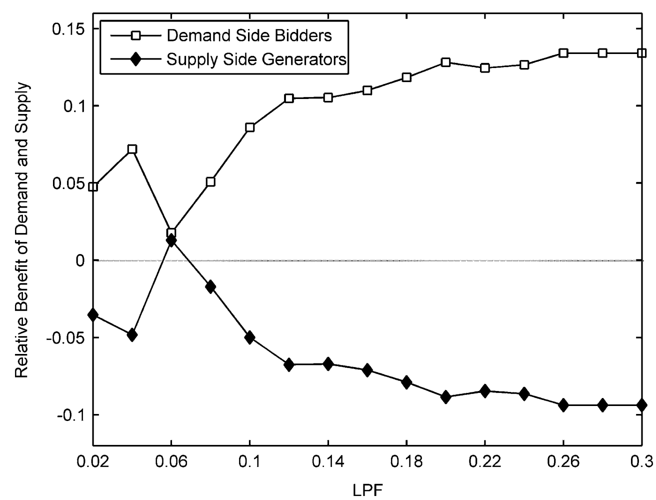


Fig. 9. Benefits and losses for demand and supply sides as a function of the demand response.

Fig. 4 shows that, integrated over the scheduling horizon, demand shifting reallocates the load in an "energy neutral" way. Typically consumers will incur some cost, or some inconvenience for shifting load, but this is not modeled. The following figures use the weighted averages defined above to summarize the effective costs (Fig. 7) and the savings (Fig. 8) of both the shifting price responsive and price taking bidders respectively. It can be observed that the effective cost of price responsive bidder drops significantly as some demand becomes price-responsive. However, Fig. 8. shows that this saving (with respect to the case without any demand shifting) diminishes as the size of the demand-shifting bid increases (i.e., as LPF increases). The small irregularities in the curve are caused by the non-convexity of the optimization problem. The fact that $\lambda_T$ is positive in most cases indicates that price-taking bidders also generally benefit from lower electricity prices as a result of the demand shifting performed by the price-responsive demand-side bidders. In essence, price-taking consumers get a partial free ride on the demand-shifting. Fig. 9. summarizes the relative benefits obtained by the demand-side bidders ($\lambda_{TD}$) and the supply side generators ($\lambda_{TG}$). This shows that the benefit to the demand-side ($\lambda_{TD}$) and the reduction of profit for the generators ($\lambda_{TG}$) both tend to increase as the demand response increases. The sum of these two quantities gives the total benefits for all market participants ($\lambda_{TA}$), which as stated in (22) is equal to the total savings in system operating cost ($\lambda_G$) (see Fig. 10). Although the generators tend to make less profit with increasing demand response, the unit commitment schedule produced by the proposed market-clearing tool will not cause any generating unit to make a loss, provided that the bid is at least equal to its actual cost.

As expected, the system is more efficient with increasing levels of demand shifting. Nevertheless, the savings in system operating cost saturate as $LPF$ increases, which is mainly due to the non-decreasing nature of the marginal production cost of the generators.

TABLE II
DEMAND SHIFTING BID VERSUS CONVENTIONAL PRICE-VOLUME BID

| | Effective Cost | Total Unsatisfied Demand |
|---|---|---|
| Price-Volume | 10.24 | 34% of total requirement |
| Demand Shifting | 10.10 | 0 |

*B. Demand Shifting Bids versus Price-Volume Bids*

This study compares the demand-shifting bids with conventional price-volume bids for demand. The bidding behavior of consumers that submit demand shifting bids are described by the following (28)–(30) and (34):

$$E^k = \frac{LPF}{K} \cdot \sum_{t=1}^{T} D_F^t \cdot \Delta t \tag{28}$$

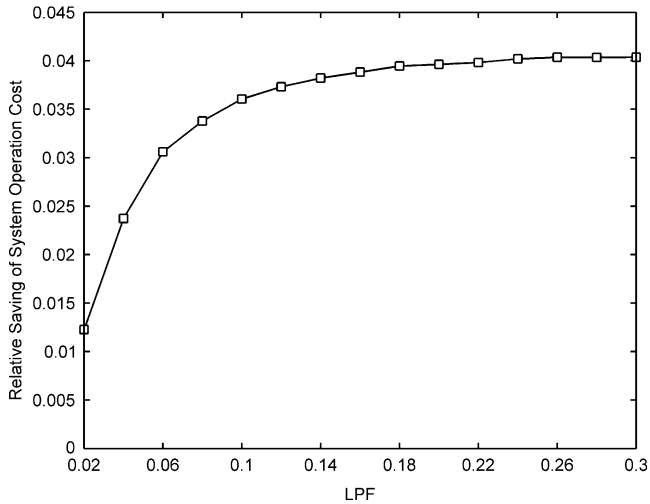


Fig. 10. Benefits obtained by the market participants as a whole as a function of the demand response.

$$D_U^{k,t} = \frac{E^k}{\Delta t} \tag{29}$$

$$D_L^{k,t} = 0. \tag{30}$$

On the other hand, the bidding behavior of consumers that submit price-volume bids are described by (28) and (31)–(34):

$$D_v^{k,t} = \frac{LPF \cdot D_F^t}{K} \tag{31}$$

where $D_v^{k,t}$ is the fixed amount of demand requested by bidder $k$ at period $t$ of a price-volume bid. Conventional price-volume bids are a special case of demand-shifting bids where the parameters are set as in (32)–(33). These constraints effectively force the demand of bidder $k$ to be either 0 or $D_v^{k,t}$

$$D_L^{k,t} = D_U^{k,t} = D_v^{k,t} \tag{32}$$

$$E^k = \sum_{t=1}^{T} D_v^{k,t} \cdot \Delta t. \tag{33}$$

The marginal benefit of the bid is the same for both types of bidders and is described by the following:

$$MB_{Sg}^{k,1,t} = \pi_L + \frac{\pi_H - \pi_L}{K-1}(k-1). \tag{34}$$

It is assumed that the value bidders place on consuming electrical energy is time invariant. These bids are also subject to constraints of the type described by (3)–(6). These equations represent a series of discrete bids that form a staircase function with a negative slope. To allow a fair comparison, the bidding prices, the amount of responsive demand, and the number of demand-side bidders for both demand-shifting and price-volume bidding mechanisms are chosen to be the same:

- $\pi_L = 10.34$ \$/MWh;
- $\pi_H = 11.24$ \$/MWh;
- $LPF = 0.05$;
- $K = 10$ bidders.

TABLE III
OFFER PRICES OF THE 10-UNIT SYSTEM.

| Unit | Minimum stable generation | Output level at elbow point 1 | 2 | Maximum capacity | Marginal cost at segment 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| 1 | 50.00 | 100.00 | 150.00 | 200.00 | 7.827 | 8.342 | 8.856 |
| 2 | 75.00 | 150.00 | 200.00 | 250.00 | 9.166 | 9.731 | 10.183 |
| 3 | 110.00 | 200.00 | 300.00 | 375.00 | 8.292 | 8.567 | 8.916 |
| 4 | 130.00 | 230.00 | 300.00 | 400.00 | 8.230 | 8.502 | 8.774 |
| 5 | 130.00 | 200.00 | 350.00 | 420.00 | 8.926 | 9.256 | 9.586 |
| 6 | 160.00 | 300.00 | 500.00 | 600.00 | 9.436 | 9.928 | 10.369 |
| 7 | 225.00 | 300.00 | 500.00 | 700.00 | 10.45 | 11.090 | 12.031 |
| 8 | 250.00 | 400.00 | 600.00 | 750.00 | 9.973 | 10.430 | 10.890 |
| 9 | 275.00 | 400.00 | 600.00 | 850.00 | 9.026 | 9.442 | 10.018 |
| 10 | 300.00 | 500.00 | 800.00 | 1000.00 | 9.927 | 10.490 | 11.057 |

With the assumptions above, we obtain ten demand bids for each bidding method, with values between 10.34 and 11.24 \$/MWh (hence the price responsive part of the system demand has an elasticity $\varepsilon$ of $-0.574$). Table II summarizes the performance of the two bidding methods and shows that if price volume bids are adopted, some of these bids will be rejected. 34% of the total energy requirement will then remain unsatisfied. On the other hand, the entire energy requirement of every demand-shifting bidder is completely satisfied and cost \$0.14 less for each MWh consumed. Therefore submitting a demand shifting bid is more beneficial as it outperforms price-volume bid in both effective cost of consumption and management of unsatisfied demand. It should be noted that results in Table II compare only the performance of the two bidding mechanisms at $\mathrm{LPF} = 0.05$. At higher levels of demand response (higher LPF), some demand shifting bids may be rejected. Nevertheless, the total unsatisfied demand with demand shifting bidding mechanism is never more than the case with price-volume bids.

## V. CONCLUSION

A day-ahead market-clearing tool that maximizes the social welfare has been presented. The tool offers consumers the opportunity to reduce their energy costs by submitting a shifting bid, provided they are flexible with the timing of their consumption. This bidding mechanism is useful in managing the risk of going unbalanced after the gate closure of a day-ahead market, especially if the day-ahead prices are volatile. The market-clearing prices tend to reduce with an increasing level of demand shifting, which benefits all bidders even if they do not participate in shifting activities. The example presented also show that demand shifting improves the economic efficiency of the day-ahead market as the effective costs of serving system demand tends to decrease. Studies have shown that demand-shifting bids outperform conventional price-volume bids in both management of unsatisfied demand and effective cost of energy consumption. It has also been observed that a substantial saving in the system operating cost is transferred to the demand-side with increasing level of demand shifting. In this regard, the extent to which the demand-side bidders are able to "game the system" by bidding strategically under the proposed auction market design is worth investigating in future research work.

TABLE IV
LOAD PROFILE FOR THE 10-UNIT SYSTEM.

| Period | Forecasted system load | Period | Forecasted system load | Period | Forecasted system load |
|---|---|---|---|---|---|
| 1 | 2025 | 9 | 3850 | 17 | 3725 |
| 2 | 2000 | 10 | 4150 | 18 | 4200 |
| 3 | 1900 | 11 | 4300 | 19 | 4300 |
| 4 | 1850 | 12 | 4400 | 20 | 3900 |
| 5 | 2025 | 13 | 4275 | 21 | 3125 |
| 6 | 2400 | 14 | 3950 | 22 | 2650 |
| 7 | 2970 | 15 | 3700 | 23 | 2300 |
| 8 | 3400 | 16 | 3550 | 24 | 2150 |

## APPENDIX

Table III lists the offer prices of the ten-unit system, and Table IV lists the load profile of the ten-unit system.

## REFERENCES

[1] R. L. Earle, "Demand elasticity in the California power exchange day-ahead market," *Elect. J.*, vol. 13, no. 8, pp. 59–65, Sep. 2000.
[2] R. H. Patrick and F. A. Wolak, "Real-time pricing and demand side participation in restructured electricity markets," in *Proc. Conf. Retail Participation in Competitive Power Markets*, 2001.
[3] C. Goldman *et al., Customer Strategies for Responding to Day-Ahead Market Hourly Electricity Pricing*. Berkeley, CA: Lawrence Berkeley National Laboratory, 2005.
[4] D. S. Kirschen, "Demand-side view of electricity markets," *IEEE Trans. Power Syst.*, vol. 18, no. 2, pp. 520–527, May 2003.
[5] S. Rassenti, V. Smith, and B. Wilson, "Controlling market power and price spikes in electricity networks: Demand-side bidding," *Proc. Nat. Acad. Sci.*, vol. 100, no. 5, pp. 2998–3003, May 2003.
[6] S. Borenstein, J. Bushnell, and F. Wolak, "Measuring market inefficiencies in California's restructured wholesale electricity market," *Amer. Econ. Rev.*, vol. 92, no. 5, pp. 1376–1405, 2002.
[7] S. Stoft, *Power System Economics: Designing Markets for Electricity*. New York: Wiley-Interscience, 2002.
[8] C.-L. Su, "Optimal demand-side participation in day-ahead electricity markets," Ph.D. dissertation, Univ. Manchester, Manchester, U.K., 2007.
[9] G. Strbac and D. Kirschen, "Assessing the competitiveness of demand-side bidding," *IEEE Trans. Power Syst.*, vol. 14, no. 1, pp. 120–125, Feb. 1999.
[10] A. Borghetti, G. Gross, and C. A. Nucci, "Auctions with explicit demand-side bidding in competitive electricity markets," in *The Next Generation of Electric Power Unit Commitment Models*. Norwell, MA: Kluwer, 2001, pp. 53–74.
[11] J. M. Arroyo and A. J. Conejo, "Multiperiod auction for a pool-based electricity market," *IEEE Trans. Power Syst.*, vol. 17, no. 4, pp. 1225–1231, Nov. 2002.
[12] J. Contreras, O. Candiles, J. I. de la Fuente, and T. Gomez, "Auction design in day-ahead electricity markets," *IEEE Trans. Power Syst.*, vol. 16, no. 3, pp. 409–417, Aug. 2001.
[13] Modeling with Xpress-MP, D. Associates, Ed., 2005. [Online]. Available: http://www.dashoptimization.com.
[14] C. Wang and S. M. Shahidehpour, "Ramp-rate limits in unit commitment and economic dispatch incorporating rotor fatigue effect," *IEEE Trans. Power Syst.*, vol. 9, no. 3, pp. 1539–1545, Aug. 1994.
[15] Y. T. Tan and D. S. Kirschen, "Co-optimization of energy and reserve in electricity markets with demand-side participation in reserve services," in *Proc. IEEE PES Power Systems Conf. Expo.*, Atlanta, GA, 2006.
[16] D. Kirschen and G. Strbac, *Fundamentals of Power System Economics*. New York: Wiley, 2004.
[17] S. Borenstein and S. Holland, "Investment efficiency in competitive electricity markets with and without time-varying retail prices," in *Center for the Study of Energy Markets Working (CSEM), No. 106*. Berkeley: Univ. California Energy Inst., 2003.

**Chua-Liang Su** (M'03) received the electrical and electronic engineer's degree from the University of Manchester Institute of Science and Technology (UMIST), Manchester, U.K., in 2003, and the Ph.D. degree from the University of Manchester in 2007.

He is currently a Research Associate at the University of Manchester.

**Daniel S. Kirschen** (M'86–SM'91–FM'07) received the electrical and mechanical engineer's degree from the Université Libre de Bruxelles, Bruxelles, Belgium, in 1979 and the M.S. and Ph.D. degrees from the University of Wisconsin-Madison in 1980 and 1985, respectively.

He is currently a Professor of electrical energy systems at the University of Manchester, Manchester, U.K.